Math AA HL Test
(No calculator)
Trigonometry (1)

--by Kevin Lin

Total mark: 80
-----/80
Time: 80 minutes

1. Consider a function $f(x)=\sin x+\cos x$
   (Where $0<x<\pi$) (7 total marks)

a): write f(x) in the form of sin*(x)*cos(a)+cos(X)*sin(a), where $a \in (0,\frac{\pi}{2})$, is a constant that needs to be determined. (4 marks)
b): Therefore, find the maximum and minimum value of the function (4 marks)

2. Determine the values of "a" such that the following equations of x has more than 1 solutions.

$$2\cos^2(x)-(a-2)\sin x+(1-a)=1$$ (Where $x \in (0,\pi)$ (7 total marks)

3. (11 marks total)
   a): show that cos(A+B) +cos(A-B)=2cosAcosB (3 marks)
   b): Therefore, show that
   $\cos(\frac{7\pi}{18})+\sin(\frac{2\pi}{9})=\cos(\frac{\pi}{10})$. (4 marks)
   c): Use the result obtained from part a), find the integral

$$\int_0^{\pi} \cos(2x-1)\cdot\cos(x-2)\,dx$$ (4 marks)

4. (9 marks total)
   a) Show the identity that $\frac{3\tan(x)-\tan^3(x)}{1-3\tan^2(x)}=\tan(3x)$ (4 marks)
   b) Hence or otherwise solve the equation. (5 marks)

$$\frac{3\tan(x)-\tan^3(x)}{4-3sec^2(x)} = \frac{2\tan x}{2-sec^2(x)}$$

5 . (8 marks total)

a): Show the identity that $\frac{\sin(4x)}{\sin(x)} = \cos(x)\cdot\cos(2x)\cdot\cos(4x)$ (4 marks)

b): Therefore, find the value of $\cos\left(\frac{\pi}{7}\right)\cdot\cos\left(\frac{2\pi}{7}\right)\cdot\cos\left(\frac{4\pi}{7}\right)$ (4 marks)

6. consider the function f(x)=ln(sin(x)+$\sqrt{3}$cos(x))-ln(4sin(x+$\frac{\pi}{3}$)).

(total 13 marks)

(Where x $\in$ [0,$\frac{\pi}{6}$))

a): write f(x) in the form of ln(a*tan(x+k)) +c, where “a”, “k”, and “c” are constants that need to be determined (5 marks)

b): Hence state the range of the function (4 marks)

c): Now there are 2 curves with equation y=x^2 and y=-1, determine how many points are there that are on 2 or more curves (4 marks)

7. (total 12 marks)

a): graph the function f(x)=4cos(x+$\frac{\pi}{6}$) for all x from 0 to 2π, make sure to label all the maxima, minima, and intercepts. (4 marks)

b): Use your graph from part a), graph the function g(x)=$\frac{3}{2\cos(x)-2\sqrt{3}\sin(x)}$ for all x from 0 to π, make sure to label all the maxima, minima, intercepts and asymptote(if there is any) (4 marks)

c): Hence, solve for all the intersections from the graph you get from b) (4 marks)

8. (total 13 marks)

Consider that the cost of product of electronic forms a sinusoidal graph depending about the market. If the market is in a good shape, it achieves its maximum value of 3000$. However, if the market situation is not very well, the cost will drop into 2000$. Assume the market starts selling its product at t=0 on its maximum value, and reaches its lowest value every 2 years statistically.

a): Find a function C(x) that describes the changing of the cost of the product (4 marks).

b) Now, consider another graph of selling price of the product. Again, it also follows a sinusoid pattern. Assume it started to sell in the lowest price of 3000$\sqrt{3}$ $ and after 2 years, it started to reach its maximum price of 4000$\sqrt{3}$$. Find the equation S(x) of the selling price. (4 marks)

c): Use your graphs from the previous questions, find the maximum and minimum value of the revenue (5 marks)